

RECEIVED: May 16, 2016

ACCEPTED: August 1, 2016

PUBLISHED: August 17, 2016

An operator basis for the Standard Model with an added scalar singlet

Ben Gripaios^a and Dave Sutherland^{a,b}

^a*Cavendish Laboratory,
J.J. Thomson Avenue, Cambridge, U.K.*

^b*Kavli Institute for Theoretical Physics,
UCSB Kohn Hall, Santa Barbara CA, U.S.A.*

E-mail: gripaios@hep.phy.cam.ac.uk, dws28@cam.ac.uk

ABSTRACT: Motivated by the possible di-gamma resonance at 750 GeV, we present a basis of effective operators for the Standard Model plus a scalar singlet at dimensions 5, 6, and 7. We point out that an earlier list at dimensions 5 and 6 contains two redundant operators at dimension 5.

KEYWORDS: Phenomenological Models, Phenomenology of Field Theories in Higher Dimensions

ARXIV EPRINT: [1604.07365](https://arxiv.org/abs/1604.07365)

Contents

1	Introduction	1
2	Method	2
3	Results	3
4	Comparison with previous results	7

1 Introduction

Both ATLAS [1] and CMS [2] have recently observed a slight excess in the $\gamma\gamma$ spectrum at an invariant mass of approximately 750 GeV. If this excess persists in the 2016 dataset, it will be apt to consider the relevant data in the most model independent way possible; any such approach will be invariably based on the number and nature of putative lagrangian terms which are consistent with the assumed symmetries of a new particle. Therefore, we present here a basis of operators up to and including mass dimension 7 for the simplest case: we assume Standard Model symmetries acting on the fields of the Standard Model plus a real scalar singlet, which we denote by ϕ . We assume flavour diagonal couplings of the fermions. At dimensions 5 and 6, a putative basis has been given recently in [3]. As we shall show, two linear combinations of the operators given there at dimension 5 are redundant (in the sense that they do not contribute to scattering amplitudes and can hence be eliminated from the basis). We also present a list of operators at dimension 7. As the large couplings required for the anomaly in $\gamma\gamma$ suggest that the cutoff scale of its effective field theory may be quite low, and as the number of basis operators at dimension 7 is still manageable, we reason that this list may also be useful.

In contrast to some recent elegant methods [4–7] using Hilbert series, we proceed by brute force, using a computer to enumerate both the possible terms consistent with given symmetries, and the redundancies between them. Although this approach taxes the computer at higher dimensions due to the exponential growth of terms and relations between them, it is tractable at the dimensions of interest, and, moreover, the code is generic and yields a wealth of explicit information on the possible choices of basis, and how to convert to or inbetween arbitrary bases, for any particular input of fields and $SU(N)$ -like symmetries. We will study these expressions systematically in a few interesting cases, including the Standard Model, in a forthcoming paper [8]. However, for now, we just summarise our approach for the Standard Model plus singlet in section 2, and then pick one basis according to simple criteria (such as minimising the number of derivatives in operators), writing it down in section 3. We compare with earlier results at dimensions 5 and 6 in section 4.

Field	Dimension	SU(3) _c	SU(2) _L	U(1) _Y	SU(2) _{lor,L}	SU(2) _{lor,R}
$L_L^{\alpha a}$	$\frac{3}{2}$	1	2	$-\frac{1}{2}$	2	1
$e_R^{\dot{\alpha}}$	$\frac{3}{2}$	1	1	-1	1	2
$Q_L^{\alpha a A}$	$\frac{3}{2}$	3	2	$\frac{1}{6}$	2	1
$u_R^{\dot{\alpha} A}$	$\frac{3}{2}$	3	1	$\frac{1}{3}$	1	2
$d_R^{\dot{\alpha} A}$	$\frac{3}{2}$	3	1	$-\frac{2}{3}$	1	2
H^a	1	1	2	$\frac{1}{2}$	1	1
$B_{(\alpha\beta)}$	2	1	1	0	3	1
$W_{b(\alpha\beta)}^a, W_{a(\alpha\beta)}^b \equiv 0$	2	1	3	0	3	1
$G_{B(\alpha\beta)}^A, G_{A(\alpha\beta)}^B \equiv 0$	2	8	1	0	3	1
ϕ	1	1	1	0	1	1

Table 1. The fields in component form, plus their representations under the Standard Model symmetries. Key: $A, B, \dots = \text{SU}(3)_c$, $a, b, \dots = \text{SU}(2)_L$, $\alpha, \beta, \dots = \text{SU}(2)_{\text{lor,L}}$, $\dot{\alpha}, \dot{\beta}, \dots = \text{SU}(2)_{\text{lor,R}}$.

2 Method

We use Python code to exhaustively generate all products of fields of a given mass dimension that transform as a singlet under the Standard Model gauge and Lorentz symmetries. Thereafter, the program enumerates all linear combinations of operators which do not contribute to scattering amplitudes at the same mass dimension order; finally, it performs some simple linear algebra to determine a suitable basis of operators to parametrise measurable effects in scattering amplitudes. We postpone a fuller exposition of the program to a forthcoming paper [8], and provide instead the following summary.

We consider the Standard Model fields, plus an additional scalar singlet, in irreps of the group $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{SO}(3,1)$. We parametrise the components of the irreps of the non-abelian $\text{SU}(N)$ gauge groups by upper indices (which transform in the fundamental irrep), lower indices (which transform in the conjugate irrep to the fundamental), and symmetric, anti-symmetric and traceless combinations thereof. Under Hermitian conjugation of the field, upper indices are lowered and *vice versa*. We treat the Lorentz group irreps as if they were the irreps of the direct product of two $\text{SU}(2)$ groups, with the exception that, under Hermitian conjugation, their indices are swapped (i.e., undotted indices are dotted and *vice versa*). As we are considering only flavour diagonal couplings, we work with just one fermion generation (flavour indices may be trivially reinserted). Table 1 displays the fields in component form.¹

The only three non-Abelian group invariants are the Kronecker delta, which contracts an upper and lower index of the same group; the lower index Levi-Civita epsilon, which

¹In this formalism, the gauge boson field strengths are three component complex fields, e.g. $B_{\alpha\beta}$. We relate them to the more familiar $B_{\mu\nu}$ via the definition $\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\beta\dot{\beta}}^{\nu}B_{\mu\nu} = -\frac{1}{2}(\epsilon_{\alpha\dot{\beta}}B_{\alpha\beta} + \epsilon_{\alpha\beta}B_{\dot{\alpha}\dot{\beta}}^{\dagger})$, wherein all other quantities follow the conventions of [9].

contracts N $SU(N)$ upper indices; and, similarly, the upper index Levi-Civita epsilon. The program generates, up to a specified dimension, all products of fields and their covariant derivatives with zero net $U(1)_Y$ charge. It then finds all possible ways of partitioning the upper and lower indices amongst epsilon and delta tensors. The resulting list of possible gauge and Lorentz singlet operators provide a hugely redundant description of measurable effects in scattering amplitudes. The program enumerates the following linear combinations of operators which make no such contribution to observables: Fierz relations (achieved by replacing products of epsilons with sums of products of deltas); integration by parts relations (by moving covariant derivatives amongst the fields of an operator); commuting derivatives (by switching the order of covariant derivatives and setting the difference equal to a sum of field strengths); and equation of motion relations, for operators of mass dimension greater than the spacetime dimension (by functionally differentiating the marginal operators and contracting the resulting equation of motion with all possible combinations of fields and derivatives).

For each linear combination the program constructs a row vector out of its coefficients, representing an unmeasurable direction in the vector space of operators. The position of each term's coefficient in the row vector corresponds to the ordering of the terms under an arbitrary comparison operator, such that undesirable terms that we hope to eliminate from the basis are to the left, and desirable terms that we wish to keep are to the right. Listed in descending priority, our preferences are for fewest derivatives, followed by fewest epsilons, followed by fewest group indices.²

The row vectors are combined into the rows of a matrix, whose rank indicates how many operators in our original list can be discarded to leave a suitable basis. The program then puts the matrix in reduced row echelon form. Each operator whose coefficient is the first non-zero entry in a row of the reduced matrix may then be consistently eliminated.

We have verified the results of the program against a few simple examples, including the model documented in appendix B of [11], as well as the one generation Standard Model at dimensions 5 [12], 6 [10], and 7 [6].

3 Results

Table 2 shows the number of independent operators at each mass dimension up to 7 for a one generation Standard Model plus a real scalar singlet. Note that we count an operator and its distinct Hermitian conjugate separately, and the entries hence reflect the number of real parameters that can be fit at each dimension, not accounting for the freedom to canonically normalise fields, nor to perform subsequent unitary/orthogonal transformations thereof.

A suitable basis of operators consists of the usual Standard Model operators at $d = 2$ and $d = 4$, the Weinberg operator at $d = 5$, the operators of tables 2 and 3 of [10]³ at $d = 6$,

²Up to simple IBP and Fierz rearrangements, this effectively reproduces the ‘Warsaw basis’ [10] in the case of the Standard Model at dimension 6.

³Excluding $Q_{qqq}^{(3)}$ and its Hermitian conjugate, which are not independent when there is only one matter generation.

d	1	2	3	4	5	6	7
SM + singlet	1	2	2	19	17	102	176
of which SM	0	1	0	16	2	84	30
of which new	1	1	2	3	15	18	146

Table 2. The number of independent operators at each dimension $d \leq 7$ that contribute to scattering amplitudes. In the last two rows we partition them according to whether they do not or do contain a ϕ field respectively.

and the operators of appendix D of [6]⁴ at $d = 7$, plus their distinct Hermitian conjugates, plus the following at each dimension $d \leq 7$ that include at least one scalar singlet ϕ .

$d = 1$ ϕ

$d = 2$ $\phi\phi$

$d = 3$ $H^\dagger_a H^a \phi$, $\phi\phi\phi$

$d = 4$ $\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}D_{\alpha\dot{\beta}}\phi D_{\beta\dot{\alpha}}\phi$, $H^\dagger_a H^a \phi\phi$, $\phi\phi\phi\phi$

$d = 5$ $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}W_{\gamma\beta b}^a W_{\alpha\delta a}^b \phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}W_{\dot{\delta}\dot{\beta} b}^{\dot{a}} W_{\dot{\alpha}\dot{\gamma} a}^{\dot{b}} \phi$, $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}G_{\gamma\beta B}^A G_{\alpha\delta A}^B \phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}G_{\dot{\delta}\dot{\beta} B}^{\dot{A}} G_{\dot{\alpha}\dot{\gamma} A}^{\dot{B}} \phi$,
 $\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{ab}H^\dagger_a Q_{LAb}^\dagger u_R^{A\dot{\alpha}} \phi$, $\epsilon_{\alpha\beta}\epsilon_{ab}H^a Q_L^{\alpha Ab} u_R^\dagger{}^\beta \phi$, $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}B_{\gamma\beta} B_{\alpha\delta} \phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}B_{\dot{\delta}\dot{\beta}}^\dagger B_{\dot{\alpha}\dot{\gamma}}^\dagger \phi$,
 $\epsilon_{\dot{\alpha}\dot{\beta}}H^a Q_{LAa}^\dagger d_R^{A\dot{\alpha}} \phi$, $\epsilon_{\alpha\beta}H^\dagger_a Q_L^{\alpha Aa} d_{RA}^\dagger{}^\beta \phi$, $\epsilon_{\alpha\beta}H^\dagger_a e_R^\dagger{}^\alpha L_L^{\beta a} \phi$, $\epsilon_{\dot{\alpha}\dot{\beta}}H^a e_R^{\dot{\beta}} L_{La}^\dagger{}^\dot{\alpha} \phi$,
 $H^\dagger_a H^\dagger_b H^a H^b \phi$, $H^\dagger_a H^a \phi\phi\phi$, $\phi\phi\phi\phi\phi$

$d = 6$ $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}B_{\dot{\delta}\dot{\beta}}^\dagger B_{\dot{\alpha}\dot{\gamma}}^\dagger \phi\phi$, $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}B_{\gamma\beta} B_{\alpha\delta} \phi\phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}G_{\dot{\delta}\dot{\beta} B}^{\dot{A}} G_{\dot{\alpha}\dot{\gamma} A}^{\dot{B}} \phi\phi$, $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}G_{\gamma\beta B}^A G_{\alpha\delta A}^B \phi\phi$,
 $H^\dagger_a H^\dagger_b H^a H^b \phi\phi$, $H^\dagger_a H^a \phi\phi\phi\phi$, $\epsilon_{\dot{\alpha}\dot{\beta}}H^a e_R^{\dot{\beta}} L_{La}^\dagger{}^\dot{\alpha} \phi\phi$, $\epsilon_{\alpha\beta}H^\dagger_a e_R^\dagger{}^\alpha L_L^{\beta a} \phi\phi$,
 $\epsilon_{\alpha\beta}H^\dagger_a Q_L^{\alpha Aa} d_{RA}^\dagger{}^\beta \phi\phi$, $\epsilon_{\dot{\alpha}\dot{\beta}}H^a Q_{LAa}^\dagger d_R^{A\dot{\alpha}} \phi\phi$, $\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{ab}H^\dagger_a Q_{LAb}^\dagger u_R^{A\dot{\alpha}} \phi\phi$,
 $\epsilon_{\alpha\beta}\epsilon_{ab}H^a Q_L^{\alpha Ab} u_R^\dagger{}^\beta \phi\phi$, $\phi\phi\phi\phi\phi\phi$, $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}W_{\gamma\beta b}^a W_{\alpha\delta a}^b \phi\phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}W_{\dot{\delta}\dot{\beta} b}^{\dot{a}} W_{\dot{\alpha}\dot{\gamma} a}^{\dot{b}} \phi\phi$,
 $\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}H^\dagger_a H^a D_{\alpha\dot{\beta}}\phi D_{\beta\dot{\alpha}}\phi$

$\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{ab}\epsilon^{de}H^\dagger_d H^\dagger_b L_{La}^\dagger L_{Le}^\dagger{}^\dot{\alpha} \phi$, $\epsilon_{\alpha\beta}\epsilon_{ab}\epsilon_{de}H^d H^b L_L^{\alpha a} L_L^{\beta e} \phi$

$d = 7$ $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}B_{\gamma\beta} B_{\alpha\delta} \phi\phi\phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}B_{\dot{\delta}\dot{\beta}}^\dagger B_{\dot{\alpha}\dot{\gamma}}^\dagger \phi\phi\phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}\epsilon^{\mu\nu}G_{\dot{\nu}\dot{\delta} C}^{\dot{A}} G_{\dot{\beta}\dot{\gamma} A}^{\dot{B}} G_{\dot{\mu}\dot{\alpha} B}^{\dot{C}} \phi$,
 $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon^{\mu\nu}G_{\mu\beta C}^A G_{\delta\nu A}^B G_{\alpha\gamma B}^C \phi$, $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}G_{\gamma\beta B}^A G_{\alpha\delta A}^B \phi\phi\phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}G_{\dot{\delta}\dot{\beta} B}^{\dot{A}} G_{\dot{\alpha}\dot{\gamma} A}^{\dot{B}} \phi\phi\phi$,
 $H^\dagger_a B_{\alpha\beta} e_R^\dagger{}^\alpha L_L^{\beta a} \phi$, $H^a B_{\dot{\alpha}\dot{\beta}}^\dagger e_R^{\dot{\beta}} L_{La}^\dagger{}^\dot{\alpha} \phi$, $H^a B_{\dot{\alpha}\dot{\beta}}^\dagger Q_{LAa}^\dagger d_R^{A\dot{\beta}} \phi$, $H^\dagger_a B_{\alpha\beta} Q_L^{\alpha Aa} d_{RA}^\dagger{}^\beta \phi$,
 $\epsilon^{ab}H^\dagger_a B_{\dot{\alpha}\dot{\beta}}^\dagger Q_{LAb}^\dagger u_R^{A\dot{\beta}} \phi$, $\epsilon_{ab}H^a B_{\alpha\beta} Q_L^{\alpha Ab} u_R^\dagger{}^\beta \phi$, $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}H^\dagger_a H^a B_{\gamma\beta} B_{\alpha\delta} \phi$,
 $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}H^\dagger_a H^a B_{\dot{\delta}\dot{\beta}}^\dagger B_{\dot{\alpha}\dot{\gamma}}^\dagger \phi$, $\epsilon^{\alpha\beta}\epsilon^{\dot{\gamma}\dot{\delta}}H^\dagger_b H^a B_{\gamma\beta} W_{\alpha\delta a}^b \phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}H^\dagger_b H^a B_{\dot{\delta}\dot{\beta}}^\dagger W_{\dot{\alpha}\dot{\gamma} a}^{\dot{b}} \phi$,
 $\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}H^\dagger_a H^a G_{\gamma\beta B}^A G_{\alpha\delta A}^B \phi$, $\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}H^\dagger_a H^a G_{\dot{\delta}\dot{\beta} B}^{\dot{A}} G_{\dot{\alpha}\dot{\gamma} A}^{\dot{B}} \phi$, $H^\dagger_a H^\dagger_b H^\dagger_a H^a H^b H^d \phi$,

⁴The operator denoted $L^3 e_c H$ should have multiplicity 1 when $N_f = 1$.

$$\begin{aligned}
 & H^\dagger_a H^\dagger_b H^a H^b \phi \phi \phi, \epsilon_{\alpha\beta} H^\dagger_a H^\dagger_b H^a e_R^\dagger{}^\alpha L_L^{\beta b} \phi, \epsilon_{\dot{\alpha}\dot{\beta}} H^\dagger_a H^a H^b e_R^{\dot{\beta}} L_L^{\dot{\alpha}} \phi, \\
 & \epsilon_{\dot{\alpha}\dot{\beta}} H^\dagger_a H^a H^b Q_{LAb}^\dagger{}^{\dot{\beta}} d_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} H^\dagger_a H^\dagger_b H^a Q_L^{\alpha Ab} d_{RA}^\dagger{}^\beta \phi, \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{ab} H^\dagger_d H^\dagger_b H^d Q_{LAa}^\dagger{}^{\dot{\beta}} u_R^{A\dot{\alpha}} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{ab} H^\dagger_d H^d H^b Q_L^{\alpha Aa} u_{RA}^\dagger{}^\beta \phi, H^\dagger_a H^a \phi \phi \phi \phi \phi, \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} H^\dagger_d H^a W_{\gamma\beta a}^b W_{\alpha\delta b}^d \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} H^\dagger_d H^a W_{\delta\beta a}^{\dot{b}} W_{\dot{\alpha}\dot{\gamma} b}^{\dot{d}} \phi, \epsilon_{\alpha\beta} H^\dagger_a e_R^\dagger{}^\alpha L_L^{\beta a} \phi \phi \phi, \epsilon_{\dot{\alpha}\dot{\beta}} H^a e_R^{\dot{\beta}} L_L^{\dot{\alpha}} \phi \phi \phi, \\
 & H^\dagger_b e_R^\dagger{}^\alpha L_L^{\beta a} W_{\alpha\beta a}^b \phi, H^a e_R^{\dot{\alpha}} L_L^{\dot{\beta}} W_{\dot{\alpha}\dot{\beta} a}^b \phi, H^\dagger_a Q_L^{\alpha Aa} d_{RB}^\dagger{}^\beta G_{\alpha\beta A}^B \phi, \\
 & H^a Q_{LBa}^\dagger{}^{\dot{\alpha}} d_R^{A\dot{\beta}} G_{\dot{\alpha}\dot{\beta} A}^B \phi, \epsilon_{\dot{\alpha}\dot{\beta}} H^a Q_{LAa}^\dagger{}^{\dot{\beta}} d_R^{A\dot{\alpha}} \phi \phi \phi, \epsilon_{\alpha\beta} H^\dagger_a Q_L^{\alpha Aa} d_{RA}^\dagger{}^\beta \phi \phi \phi, \\
 & H^\dagger_b Q_L^{\alpha Aa} d_{RA}^\dagger{}^\beta W_{\alpha\beta a}^b \phi, H^a Q_{LAb}^\dagger{}^{\dot{\beta}} d_R^{A\dot{\beta}} W_{\dot{\alpha}\dot{\beta} a}^b \phi, \epsilon_{ab} H^a Q_L^{\alpha Ab} u_{RB}^\dagger{}^\beta G_{\alpha\beta A}^B \phi, \\
 & \epsilon^{ab} H^\dagger_a Q_{LBb}^\dagger{}^{\dot{\alpha}} u_R^{A\dot{\beta}} G_{\dot{\alpha}\dot{\beta} A}^B \phi, \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{ab} H^\dagger_a Q_{LAb}^\dagger{}^{\dot{\beta}} u_R^{A\dot{\alpha}} \phi \phi \phi, \epsilon_{\alpha\beta} \epsilon_{ab} H^a Q_L^{\alpha Ab} u_{RA}^\dagger{}^\beta \phi \phi \phi, \\
 & \epsilon_{ab} H^a Q_L^{\alpha Ad} u_{RA}^\dagger{}^\beta W_{\alpha\beta d}^b \phi, \epsilon^{ab} H^\dagger_a Q_{LAa}^\dagger{}^{\dot{\alpha}} u_R^{A\dot{\beta}} W_{\dot{\alpha}\dot{\beta} b}^d \phi, \phi \phi \phi \phi \phi \phi \phi, \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} W_{\gamma\beta b}^a W_{\alpha\delta a}^b \phi \phi \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} W_{\delta\beta b}^{\dot{a}} W_{\dot{\alpha}\dot{\gamma} a}^{\dot{b}} \phi \phi \phi, \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} \epsilon^{\mu\nu} W_{\dot{\nu}\dot{\delta} d}^{\dot{a}} W_{\dot{\beta}\dot{\gamma} a}^{\dot{b}} W_{\dot{\mu}\dot{\alpha} b}^{\dot{d}} \phi, \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} \epsilon^{\mu\nu} W_{\mu\beta d}^a W_{\delta\nu a}^b W_{\alpha\gamma b}^d \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} d_{RA}^\dagger{}^\alpha d_{RB}^\dagger{}^\beta d_R^{A\dot{\beta}} d_R^{B\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} e_R^\dagger{}^\alpha e_R^{\dot{\beta}} d_{RA}^\dagger{}^\beta d_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} e_R^\dagger{}^\alpha e_R^{\dot{\beta}} e_R^{\dot{\beta}} e_R^{\dot{\alpha}} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} e_R^\dagger{}^\alpha e_R^{\dot{\beta}} L_{La}^\dagger{}^{\dot{\alpha}} L_L^{\beta a} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} e_R^\dagger{}^\alpha e_R^{\dot{\beta}} Q_{LAa}^\dagger{}^{\dot{\beta}} Q_L^{\beta Aa} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} e_R^\dagger{}^\alpha e_R^{\dot{\beta}} u_{RA}^\dagger{}^\beta u_R^{A\dot{\alpha}} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} e_R^\dagger{}^\alpha L_L^{\beta a} Q_{LAa}^\dagger{}^{\dot{\beta}} d_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} e_R^{\dot{\beta}} L_{La}^\dagger{}^{\dot{\alpha}} Q_L^{\alpha Aa} d_{RA}^\dagger{}^\beta \phi, \\
 & \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{\dot{\gamma}\dot{\delta}} \epsilon^{ab} e_R^{\dot{\delta}} L_{La}^\dagger{}^{\dot{\beta}} Q_{LAb}^\dagger{}^{\dot{\gamma}} u_R^{A\dot{\alpha}} \phi, \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{\dot{\gamma}\dot{\delta}} \epsilon^{ab} e_R^{\dot{\delta}} L_{La}^\dagger{}^{\dot{\beta}} Q_{LAb}^\dagger{}^{\dot{\gamma}} u_R^{A\dot{\alpha}} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \epsilon_{ab} e_R^\dagger{}^\alpha L_L^{\delta a} Q_L^{\beta Ab} u_{RA}^\dagger{}^\gamma \phi, \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \epsilon_{ab} e_R^\dagger{}^\alpha L_L^{\beta a} Q_L^{\delta Ab} u_{RA}^\dagger{}^\gamma \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} L_{La}^\dagger{}^{\dot{\beta}} L_L^{\alpha a} d_{RA}^\dagger{}^\beta d_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} L_{La}^\dagger{}^{\dot{\beta}} L_{Lb}^\dagger{}^{\dot{\alpha}} L_L^{\alpha a} L_L^{\beta b} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} L_{La}^\dagger{}^{\dot{\beta}} L_L^{\alpha a} Q_{LAb}^\dagger{}^{\dot{\alpha}} Q_L^{\beta Ab} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} L_{Lb}^\dagger{}^{\dot{\beta}} L_L^{\alpha a} Q_{LAa}^\dagger{}^{\dot{\alpha}} Q_L^{\beta Ab} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} L_{La}^\dagger{}^{\dot{\beta}} L_L^{\alpha a} u_{RA}^\dagger{}^\beta u_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} Q_{LAa}^\dagger{}^{\dot{\beta}} Q_L^{\alpha Aa} d_{RB}^\dagger{}^\beta d_R^{B\dot{\alpha}} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} Q_{LBa}^\dagger{}^{\dot{\beta}} Q_L^{\alpha Aa} d_{RA}^\dagger{}^\beta d_R^{B\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} Q_{LAa}^\dagger{}^{\dot{\beta}} Q_{LBb}^\dagger{}^{\dot{\alpha}} Q_L^{\alpha Aa} Q_L^{\beta Bb} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} Q_{LBa}^\dagger{}^{\dot{\beta}} Q_{LAB}^\dagger{}^{\dot{\alpha}} Q_L^{\alpha Aa} Q_L^{\beta Bb} \phi, \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{\dot{\gamma}\dot{\delta}} \epsilon^{ab} Q_{LAa}^\dagger{}^{\dot{\delta}} Q_{LBb}^\dagger{}^{\dot{\gamma}} u_R^{A\dot{\beta}} d_R^{B\dot{\alpha}} \phi, \\
 & \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{\dot{\gamma}\dot{\delta}} \epsilon^{ab} Q_{LBa}^\dagger{}^{\dot{\delta}} Q_{LAB}^\dagger{}^{\dot{\beta}} u_R^{A\dot{\alpha}} d_R^{B\dot{\gamma}} \phi, \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \epsilon_{ab} Q_L^{\alpha Aa} Q_L^{\beta Bb} u_{RA}^\dagger{}^\delta d_{RB}^\dagger{}^\gamma \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \epsilon_{ab} Q_L^{\gamma Aa} Q_L^{\beta Bb} u_{RB}^\dagger{}^\alpha d_{RA}^\dagger{}^\delta \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} Q_{LAa}^\dagger{}^{\dot{\beta}} Q_L^{\alpha Aa} u_{RB}^\dagger{}^\beta u_R^{B\dot{\alpha}} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} Q_{LBa}^\dagger{}^{\dot{\beta}} Q_L^{\alpha Aa} u_{RA}^\dagger{}^\beta u_R^{B\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} u_{RA}^\dagger{}^\alpha u_R^{A\dot{\beta}} d_{RB}^\dagger{}^\beta d_R^{B\dot{\alpha}} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} u_{RB}^\dagger{}^\alpha u_R^{A\dot{\beta}} d_{RA}^\dagger{}^\beta d_R^{B\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} u_{RA}^\dagger{}^\alpha u_{RB}^\dagger{}^\beta u_R^{A\dot{\beta}} u_R^{B\dot{\alpha}} \phi, \epsilon^{\alpha\beta} B_{\gamma\beta} D_{\alpha\dot{\alpha}} d_{RA}^\dagger{}^\gamma d_R^{A\dot{\alpha}} \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} B_{\dot{\beta}\dot{\gamma}} d_{RA}^\dagger{}^{\dot{\alpha}} D_{\alpha\dot{\alpha}} d_R^{A\dot{\gamma}} \phi, \epsilon^{\alpha\beta} B_{\gamma\beta} D_{\alpha\dot{\alpha}} e_R^\dagger{}^\gamma e_R^{\dot{\alpha}} \phi, \epsilon^{\dot{\alpha}\dot{\beta}} B_{\dot{\beta}\dot{\gamma}} e_R^\dagger{}^{\dot{\alpha}} D_{\alpha\dot{\alpha}} e_R^{\dot{\gamma}} \phi, \\
 & \epsilon^{\alpha\beta} B_{\gamma\beta} L_{La}^\dagger{}^{\dot{\alpha}} D_{\alpha\dot{\alpha}} L_L^{\gamma a} \phi, \epsilon^{\dot{\alpha}\dot{\beta}} B_{\dot{\beta}\dot{\gamma}} D_{\alpha\dot{\alpha}} L_{La}^\dagger{}^{\dot{\gamma}} L_L^{\alpha a} \phi, \epsilon^{\alpha\beta} B_{\gamma\beta} Q_{LAa}^\dagger{}^{\dot{\alpha}} D_{\alpha\dot{\alpha}} Q_L^{\gamma Aa} \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} B_{\dot{\beta}\dot{\gamma}} D_{\alpha\dot{\alpha}} Q_{LAa}^\dagger{}^{\dot{\gamma}} Q_L^{\alpha Aa} \phi, \epsilon^{\alpha\beta} B_{\gamma\beta} D_{\alpha\dot{\alpha}} u_{RA}^\dagger{}^\gamma u_R^{A\dot{\alpha}} \phi, \epsilon^{\dot{\alpha}\dot{\beta}} B_{\dot{\beta}\dot{\gamma}} u_{RA}^\dagger{}^{\dot{\alpha}} D_{\alpha\dot{\alpha}} u_R^{A\dot{\gamma}} \phi, \\
 & D_{\alpha\dot{\alpha}} H^\dagger_a H^a d_{RA}^\dagger{}^{\dot{\alpha}} d_R^{A\dot{\alpha}} \phi, H^\dagger_a D_{\alpha\dot{\alpha}} H^a d_{RA}^\dagger{}^{\dot{\alpha}} d_R^{A\dot{\alpha}} \phi, D_{\alpha\dot{\alpha}} H^\dagger_a H^a e_R^\dagger{}^{\dot{\alpha}} e_R^{\dot{\alpha}} \phi, \\
 & H^\dagger_a D_{\alpha\dot{\alpha}} H^a e_R^\dagger{}^{\dot{\alpha}} e_R^{\dot{\alpha}} \phi, D_{\alpha\dot{\alpha}} H^\dagger_a H^a L_{Lb}^\dagger{}^{\dot{\alpha}} L_L^{\alpha b} \phi, D_{\alpha\dot{\alpha}} H^\dagger_b H^a L_{La}^\dagger{}^{\dot{\alpha}} L_L^{\alpha b} \phi, \\
 & H^\dagger_a D_{\alpha\dot{\alpha}} H^a L_{Lb}^\dagger{}^{\dot{\alpha}} L_L^{\alpha b} \phi, H^\dagger_b D_{\alpha\dot{\alpha}} H^a L_{La}^\dagger{}^{\dot{\alpha}} L_L^{\alpha b} \phi, D_{\alpha\dot{\alpha}} H^\dagger_a H^a Q_{LAb}^\dagger{}^{\dot{\alpha}} Q_L^{\alpha Ab} \phi, \\
 & D_{\alpha\dot{\alpha}} H^\dagger_b H^a Q_{LAa}^\dagger{}^{\dot{\alpha}} Q_L^{\alpha Ab} \phi, H^\dagger_a D_{\alpha\dot{\alpha}} H^a Q_{LAb}^\dagger{}^{\dot{\alpha}} Q_L^{\alpha Ab} \phi, H^\dagger_b D_{\alpha\dot{\alpha}} H^a Q_{LAa}^\dagger{}^{\dot{\alpha}} Q_L^{\alpha Ab} \phi,
 \end{aligned}$$

$$\begin{aligned}
 & \epsilon_{ab} D_{\alpha\dot{\alpha}} H^a H^b u_{RA}^{\dagger\alpha} d_R^{A\dot{\alpha}} \phi, \epsilon^{ab} D_{\alpha\dot{\alpha}} H_a^\dagger H_b^\dagger u_R^{A\dot{\alpha}} d_{RA}^{\dagger\alpha} \phi, D_{\alpha\dot{\alpha}} H_a^\dagger H^a u_{RA}^{\dagger\alpha} u_R^{A\dot{\alpha}} \phi, \\
 & H_a^\dagger D_{\alpha\dot{\alpha}} H^a u_{RA}^{\dagger\alpha} u_R^{A\dot{\alpha}} \phi, \epsilon^{\alpha\beta} D_{\beta\dot{\alpha}} d_{RB}^{\dagger\gamma} d_R^{A\dot{\alpha}} G_{\gamma\alpha A}^B, \epsilon^{\dot{\alpha}\dot{\beta}} d_{RB}^{\dagger\alpha} D_{\alpha\dot{\beta}} d_R^{A\dot{\gamma}} G_{\dot{\alpha}\dot{\gamma}A}^{\dot{\beta}B}, \\
 & \epsilon^{\alpha\beta} L_{Lb}^{\dagger\dot{\alpha}} D_{\beta\dot{\alpha}} L_L^{\gamma a} W_{\gamma\alpha a}^b \phi, \epsilon^{\dot{\alpha}\dot{\beta}} D_{\alpha\dot{\beta}} L_{Lb}^{\dagger\gamma} L_L^{\alpha a} W_{\dot{\alpha}\gamma a}^{\dot{\beta}b} \phi, \epsilon^{\alpha\beta} Q_{L Ba}^{\dagger\dot{\alpha}} D_{\beta\dot{\alpha}} Q_L^{\gamma Aa} G_{\gamma\alpha A}^B \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} D_{\alpha\dot{\beta}} Q_{L Ba}^{\dagger\gamma} Q_L^{\alpha Aa} G_{\dot{\alpha}\gamma A}^{\dot{\beta}B}, \epsilon^{\alpha\beta} Q_{L Ab}^{\dagger\dot{\alpha}} D_{\beta\dot{\alpha}} Q_L^{\gamma Aa} W_{\gamma\alpha a}^b \phi, \epsilon^{\dot{\alpha}\dot{\beta}} D_{\alpha\dot{\beta}} Q_{L Ab}^{\dagger\gamma} Q_L^{\alpha Aa} W_{\dot{\alpha}\gamma a}^{\dot{\beta}b} \phi, \\
 & \epsilon^{\alpha\beta} D_{\beta\dot{\alpha}} u_{RB}^{\dagger\gamma} u_R^{A\dot{\alpha}} G_{\gamma\alpha A}^B, \epsilon^{\dot{\alpha}\dot{\beta}} u_{RB}^{\dagger\alpha} D_{\alpha\dot{\beta}} u_R^{A\dot{\gamma}} G_{\dot{\alpha}\gamma A}^{\dot{\beta}B}, \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} \epsilon^{\dot{\alpha}\dot{\beta}} D_{\delta\dot{\beta}} H_a^\dagger D_{\alpha\dot{\alpha}} H^a B_{\beta\gamma} \phi, \\
 & \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} D_{\alpha\dot{\delta}} H_a^\dagger D_{\beta\dot{\beta}} H^a B_{\dot{\alpha}\dot{\gamma}}^\dagger \phi, \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} H_a^\dagger H_b^\dagger D_{\alpha\dot{\beta}} H^a D_{\beta\dot{\alpha}} H^b \phi, \\
 & \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} D_{\alpha\dot{\beta}} H_a^\dagger D_{\beta\dot{\alpha}} H_a^\dagger H^a H^b \phi, \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} D_{\alpha\dot{\beta}} H_a^\dagger H_b^\dagger D_{\beta\dot{\alpha}} H^a H^b \phi, \\
 & \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} D_{\alpha\dot{\beta}} H_a^\dagger D_{\beta\dot{\alpha}} H^a \phi \phi, \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} \epsilon^{\dot{\alpha}\dot{\beta}} D_{\delta\dot{\beta}} H_a^\dagger D_{\alpha\dot{\alpha}} H^a W_{\beta\gamma a}^b \phi, \\
 & \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} D_{\alpha\dot{\delta}} H_a^\dagger D_{\beta\dot{\beta}} H^a W_{\dot{\alpha}\dot{\gamma}a}^{\dot{\beta}b} \phi, \epsilon^{\dot{\alpha}\dot{\beta}} H_a^\dagger D_{\beta\dot{\beta}} e_R^{\dagger\alpha} D_{\alpha\dot{\alpha}} L_L^{\beta a} \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} H_a^\dagger D_{\beta\dot{\beta}} e_R^{\dagger\alpha} D_{\alpha\dot{\alpha}} L_L^{\beta a} \phi, \epsilon^{\alpha\beta} H^a D_{\alpha\dot{\beta}} e_R^{\dot{\alpha}} D_{\beta\dot{\alpha}} L_{La}^{\dagger\dot{\beta}} \phi, \epsilon^{\alpha\beta} H^a D_{\alpha\dot{\beta}} e_R^{\dot{\alpha}} D_{\beta\dot{\alpha}} L_{La}^{\dagger\dot{\beta}} \phi, \\
 & \epsilon^{\alpha\beta} H^a D_{\alpha\dot{\beta}} Q_{L Aa}^{\dagger\dot{\alpha}} D_{\beta\dot{\alpha}} d_R^{A\dot{\beta}} \phi, \epsilon^{\alpha\beta} H^a D_{\alpha\dot{\beta}} Q_{L Aa}^{\dagger\dot{\alpha}} D_{\beta\dot{\alpha}} d_R^{A\dot{\beta}} \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} H_a^\dagger D_{\beta\dot{\beta}} Q_L^{\alpha Aa} D_{\alpha\dot{\alpha}} d_{RA}^{\dagger\beta} \phi, \epsilon^{\dot{\alpha}\dot{\beta}} H_a^\dagger D_{\beta\dot{\beta}} Q_L^{\alpha Aa} D_{\alpha\dot{\alpha}} d_{RA}^{\dagger\beta} \phi, \\
 & \epsilon^{\alpha\beta} \epsilon^{ab} H_a^\dagger D_{\alpha\dot{\beta}} Q_{L Ab}^{\dagger\dot{\alpha}} D_{\beta\dot{\alpha}} u_R^{A\dot{\beta}} \phi, \epsilon^{\alpha\beta} \epsilon^{ab} H_a^\dagger D_{\alpha\dot{\beta}} Q_{L Ab}^{\dagger\dot{\alpha}} D_{\beta\dot{\alpha}} u_R^{A\dot{\beta}} \phi, \\
 & \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon_{ab} H^a D_{\beta\dot{\beta}} Q_L^{\alpha Ab} D_{\alpha\dot{\alpha}} u_{RA}^{\dagger\beta} \phi, \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon_{ab} H^a D_{\beta\dot{\beta}} Q_L^{\alpha Ab} D_{\alpha\dot{\alpha}} u_{RA}^{\dagger\beta} \phi
 \end{aligned}$$

$$\begin{aligned}
 & \epsilon_{\alpha\beta} \epsilon_{ab} \epsilon_{de} H^d H^b L_L^{\alpha a} L_L^{\beta e} \phi \phi, \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{ab} \epsilon^{de} H_a^\dagger H_b^\dagger L_{La}^{\dagger\dot{\beta}} L_{Le}^{\dagger\dot{\alpha}} \phi \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{ab} \epsilon_{ABC} e_R^{\dot{\beta}} Q_L^{\alpha Ba} Q_L^{\beta Cb} u_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{ab} \epsilon^{ABC} e_R^{\dagger\alpha} Q_{L Aa}^{\dagger\dot{\beta}} Q_{L Bb}^{\dagger\dot{\alpha}} u_{RC}^{\dagger\beta} \phi, \\
 & \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} \epsilon_{ABC} e_R^{\dot{\delta}} u_R^{B\dot{\beta}} u_R^{C\dot{\gamma}} d_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \epsilon^{ABC} e_R^{\dagger\alpha} u_{RA}^{\dagger\delta} u_{RB}^{\dagger\beta} d_{RC}^{\dagger\gamma} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \epsilon_{ab} \epsilon_{de} \epsilon_{ABC} L_L^{\gamma a} Q_L^{\beta Be} Q_L^{\alpha Cb} Q_L^{\delta Ad} \phi, \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} \epsilon^{ab} \epsilon^{de} \epsilon^{ABC} L_{La}^{\dagger\dot{\delta}} Q_{L Ae}^{\dagger\dot{\beta}} Q_{L Bb}^{\dagger\dot{\alpha}} Q_{L Cd}^{\dagger\gamma} \phi, \\
 & \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{ab} \epsilon_{ABC} L_L^{\alpha a} Q_L^{\beta Bb} u_R^{C\dot{\beta}} d_R^{A\dot{\alpha}} \phi, \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{ab} \epsilon^{ABC} L_{La}^{\dagger\dot{\beta}} Q_{L Ab}^{\dagger\dot{\alpha}} u_{RB}^{\dagger\alpha} d_{RC}^{\dagger\beta} \phi
 \end{aligned}$$

Upper case Latin indices transform under $SU(3)_c$, lower case Latin under $SU(2)_L$, undotted Greek under the left-handed part of the Lorentz group, and dotted Greek indices under the right-handed part. Horizontal lines separate operators that conserve both B and L , and those that don't. Note that the inclusion of a real scalar singlet spoils the correspondence between operator dimension and B and L charges which is present in the Standard Model [13, 14].

We use colours to denote the effects of requiring that the lagrangian respect CP, in the two cases that ϕ is CP even and CP odd. Pairs of adjacent operators in blue are Hermitian conjugates of each other (up to a sign): requiring CP invariance, these operators must have equal real coefficients (up to the same sign). Pairs of adjacent operators in green or orange are also Hermitian conjugates of each other (up to a sign): requiring CP invariance, these operators must have either equal real or equal imaginary coefficients (up to the same sign) in the respective cases of ϕ CP even and CP odd when green, or in the respective cases of ϕ CP odd and CP even when orange. Red operators are simply forbidden if we require CP invariance and that ϕ be CP odd.

4 Comparison with previous results

At dimension 5, ref. [3] has two extra operators in their basis. In our notation, these may be written as $\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\phi D_{\alpha\dot{\beta}}\phi D_{\beta\dot{\alpha}}\phi$ and $\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\phi D_{\alpha\dot{\beta}}H^\dagger_a D_{\beta\dot{\alpha}}H^a$. To see that the first is redundant, we first integrate by parts to get a ‘ $\phi^2\Box\phi$ ’ term and then use the equation of motion for ϕ to obtain a sum of products of ϕ and H fields, without derivatives. To see that the second is redundant, first integrate by parts to obtain a sum of three terms: ‘ $\phi H^\dagger D^2 H$ ’, ‘ $\Box\phi H^\dagger H$ ’, and ‘ $\partial_\mu\phi H^\dagger \overleftrightarrow{D}^\mu H$ ’. The former two operators may be expressed in terms of others in the basis using the equations of motion for the Higgs and singlet respectively. The latter term, via the B ’s equation of motion, may be expressed as a sum of ‘ $\partial_\mu\phi\bar{\Psi}\gamma^\mu\Psi$ ’-like terms, where Ψ is a generic fermion. Finally, integrating by parts again, and using the fermions’ equations of motion, we obtain a sum of terms already in the basis.

Note that, at dimension 6, ref. [3] opt to exclude the lepton-number violating operator obtained by multiplying the Weinberg operator [12] in the SM at dimension 5 by ϕ , viz. $\epsilon_{\alpha\beta}\epsilon_{ab}\epsilon_{de}H^d H^b L_L^{\alpha a} L_L^{\beta e}\phi$ and its Hermitian conjugate.

Acknowledgments

We thank T. Melia, M. Bauer and M. Neubert for pointing out errors in an earlier version. BG acknowledges the support of STFC (grant ST/L000385/1) and King’s College, Cambridge. DS also acknowledges the support of STFC, and Emmanuel College, Cambridge, and thanks KITP for hospitality. This research was supported in part by the National Science Foundation under Grant No. PHY11-25915.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] ATLAS collaboration, *Search for resonances decaying to photon pairs in 3.2 fb⁻¹ of pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, [ATLAS-CONF-2015-081](#).
- [2] CMS collaboration, *Search for new physics in high mass diphoton events in proton-proton collisions at $\sqrt{s} = 13$ TeV*, [CMS-PAS-EXO-15-004](#).
- [3] R. Franceschini et al., *Digamma, what next?*, *JHEP* **07** (2016) 150 [[arXiv:1604.06446](#)] [[INSPIRE](#)].
- [4] L. Lehman and A. Martin, *Hilbert Series for Constructing Lagrangians: expanding the phenomenologist’s toolbox*, *Phys. Rev. D* **91** (2015) 105014 [[arXiv:1503.07537](#)] [[INSPIRE](#)].
- [5] B. Henning, X. Lu, T. Melia and H. Murayama, *Hilbert series and operator bases with derivatives in effective field theories*, [arXiv:1507.07240](#) [[INSPIRE](#)].
- [6] L. Lehman and A. Martin, *Low-derivative operators of the Standard Model effective field theory via Hilbert series methods*, *JHEP* **02** (2016) 081 [[arXiv:1510.00372](#)] [[INSPIRE](#)].
- [7] B. Henning, X. Lu, T. Melia and H. Murayama, 2, 84, 30, 993, 560, 15456, 11962, 261485, ...: *Higher dimension operators in the SM EFT*, [arXiv:1512.03433](#) [[INSPIRE](#)].

- [8] B. Gripaios and D. Sutherland, *Investigating operator bases with a computer*, to appear.
- [9] H.K. Dreiner, H.E. Haber and S.P. Martin, *Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry*, *Phys. Rept.* **494** (2010) 1 [[arXiv:0812.1594](#)] [[INSPIRE](#)].
- [10] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [[arXiv:1008.4884](#)] [[INSPIRE](#)].
- [11] M.B. Einhorn and J. Wudka, *The Bases of Effective Field Theories*, *Nucl. Phys. B* **876** (2013) 556 [[arXiv:1307.0478](#)] [[INSPIRE](#)].
- [12] S. Weinberg, *Baryon- and Lepton-Nonconserving Processes*, *Phys. Rev. Lett.* **43** (1979) 1566 [[INSPIRE](#)].
- [13] A. de Gouvêa, J. Herrero-Garcia and A. Kobach, *Neutrino Masses, Grand Unification and Baryon Number Violation*, *Phys. Rev. D* **90** (2014) 016011 [[arXiv:1404.4057](#)] [[INSPIRE](#)].
- [14] A. Kobach, *Baryon Number, Lepton Number and Operator Dimension in the Standard Model*, *Phys. Lett. B* **758** (2016) 455 [[arXiv:1604.05726](#)] [[INSPIRE](#)].